ENS DE LYON

Numerical frugality in optimization: Newton's methods in mixed precision

EUROPT 2026, Linz

G. Carrino, N. Brisebarre, T. Mary, E. Riccietti | Thursday 9th July, 2026

Joint work with

N. Brisebarre

T. Mary

E. Riccietti

Problem statement

Optimization problem

Solving the following optimization problem:

$$\min_{x \in \mathbb{R}^n} f(x)$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a smooth function.

A second-order optimizer

We want to solve this using **Newton's method**:

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3. Update $x_{i+1} = x_i + d_i$

Newton's cost

Newton's method comes with two main sources of cost…

Solving the linear system

$$H(x_i)\, d_i = -g_i$$

Computing the Hessian

$$H(x_i)$$

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$$\downarrow$$

inexact Newton methods

$$\|H(x_i)d_i + g_i\| \leq \eta \|g_i\|$$

Computing the Hessian

$$H(x_i)$$

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quasi-Newton methods

$$B(x) \approx H(x)$$

…but many possible approximations!

Quasi-Newton: an example

When solving least-squares problems,

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$$\min_{x \in \mathbb{R}^n} \frac{1}{2} \|R(x)\|^2, \quad R : \mathbb{R}^n \to \mathbb{R}^m,$$

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Gauss-Newton discards this!

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- ***Idea***: use **low precision floating-point arithmetic** to reduce the overall cost.

Floating-point precision

$$\mathsf{fl}(x \circ y) = (x \circ y)(1+\delta), \quad |\delta| \leq u, \quad \circ \in \{+, -, *, /\}$$

Floating-point arithmetic model.

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Floating-point arithmetic model.

Type	Size	Unit roundoff (u)	Approx. range
bfloat16	16 bits	4×10^{-3}	$10^{\pm 38}$
fp16	16 bits	5×10^{-4}	$10^{\pm 4}$
fp32	32 bits	6×10^{-8}	$10^{\pm 38}$
fp64	64 bits	1×10^{-16}	$10^{\pm 308}$

$\approx 30\times$

$\approx 4\times$

Floating point formats.

Reducing the cost

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- ***Challenge***: how to do it without sacrificing convergence?

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We can leverage **mixed precision**!

Mixed precision Newton's method

Error model

For each iteration i, the mixed precision Newton's step satisfies

$$\widehat{d}_i \coloneqq -\left(H(\widehat{x}_i) + E_i^H\right)^{-1}\left(g(\widehat{x}_i) + e_i^g\right),$$
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- e_i^g: inexact gradient computations.
- e_i^+: errors in the update step.

Local convergence analysis

$$\|\widehat{x}_{i+1} - x^*\| \leq \alpha_i \|\widehat{x}_i - x^*\|^2 + \beta_i \|\widehat{x}_i - x^*\| + \gamma_i,$$

Mixed precision Newton's method convergence rate.

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Mixed precision Newton's method convergence rate.

- α_i: standard Newton's quadratic convergence;
- β_i: linear convergence influenced by Hessian errors;
- γ_i: **limiting accuracy** mainly related to gradient errors.

Error model applications

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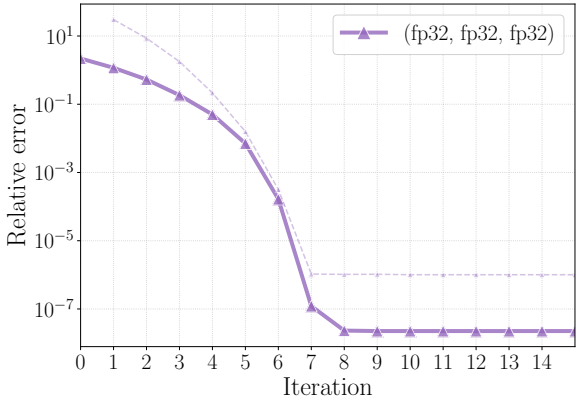
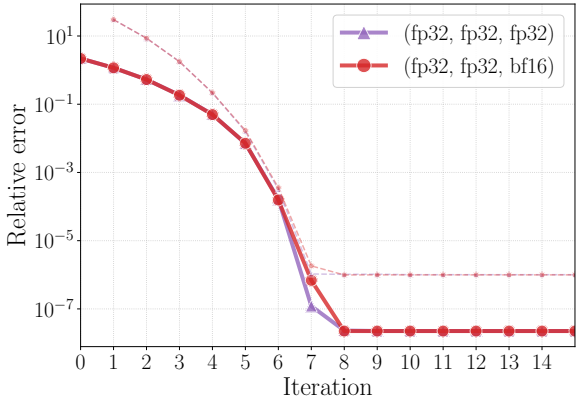
We will denote precision sets as (u_g, u, u_H)

Charts - Mixed precision Newton



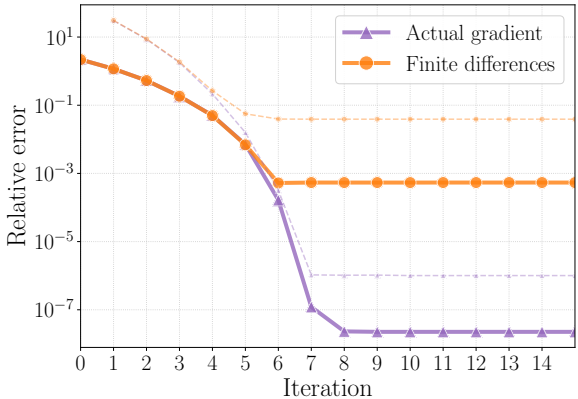
ENGVAL1. Mixed precision Newton relative error and its predicted rate.

Charts - Two mixed precision settings



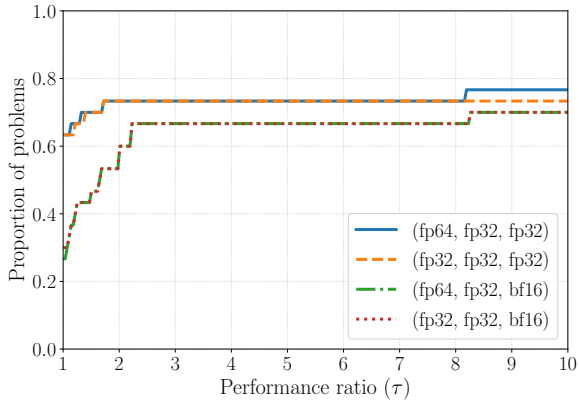
ENGVAL1. Different mixed precision settings relative error.

Charts - Finite differences



ENGVAL1. Finite differences vs true gradient relative error.

Charts - CUTEst performance profile



`CUTEst problems collection`. Performance profile of different mixed precision settings.

Error model extensions

Inexact Newton

$$\|H(x_i)d_i + g_i\| \leq \eta\|g_i\|$$

linear solver stopped early at tolerance η

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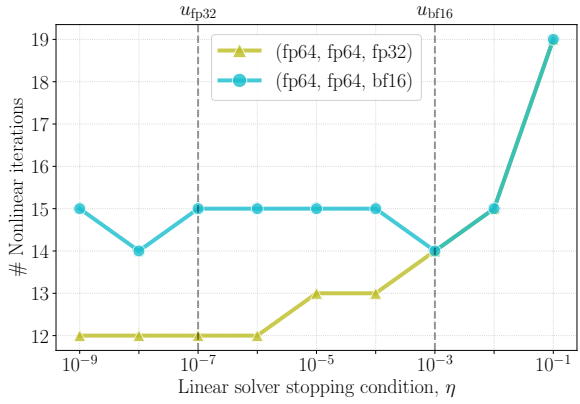
Charts - Inexact Newton



ENGVAL1. Inexact Newton: iterations vs. η.

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Gauss-Newton

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Hessian approximation discarding $S(\widehat{x}_i)$

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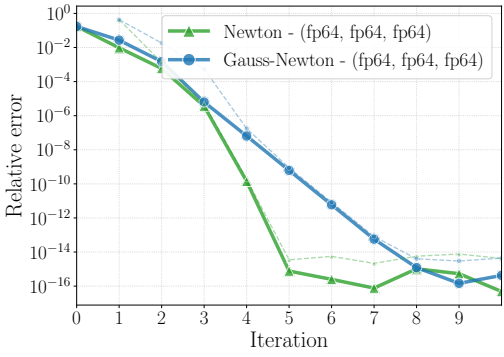

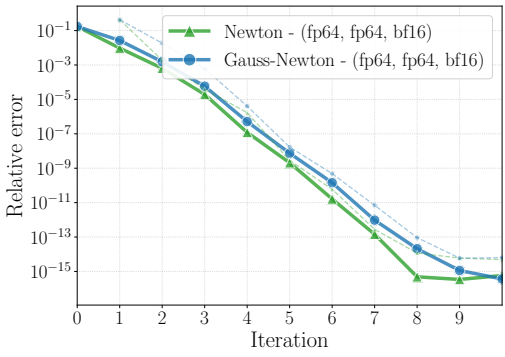

SINREG. Gauss-Newton vs. Newton's method.

Contributions

Our contribution

01 **Error model** — A comprehensive framework for mixed precision Newton's methods.

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02 **Convergence analysis** — Rigorous study of the interplay between *precision* and *convergence rate*.

03 **Numerical validation** — Experiments confirming the practical benefits of mixed precision strategies.

Short-term directions

Scaling strategies

Scaled values allowing for lower precision linear solvers.

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HPC benchmarking

Practical implementations for large-scale optimization.

New perspectives

Towards machine learning

Mixed precision Newton's method

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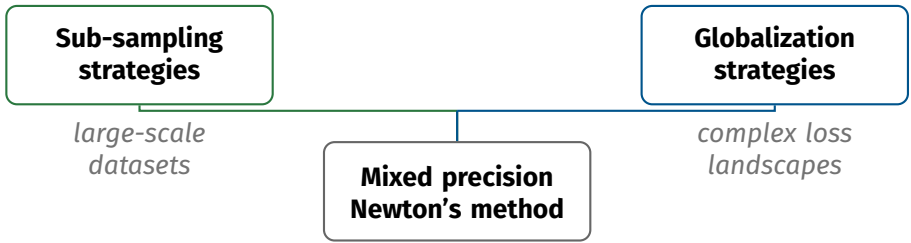
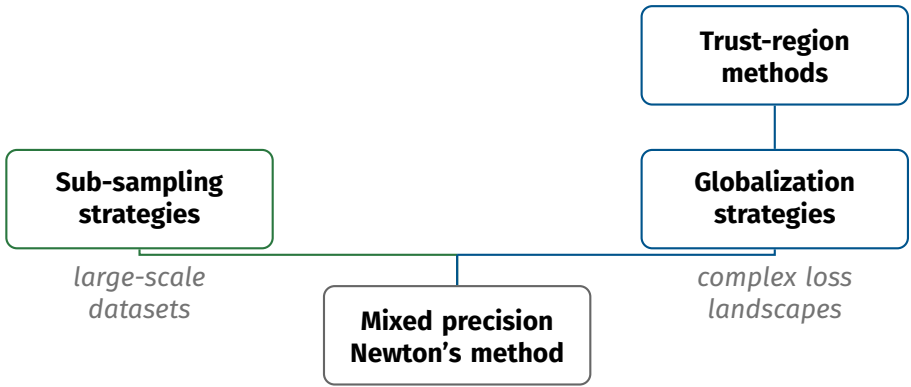


Towards machine learning



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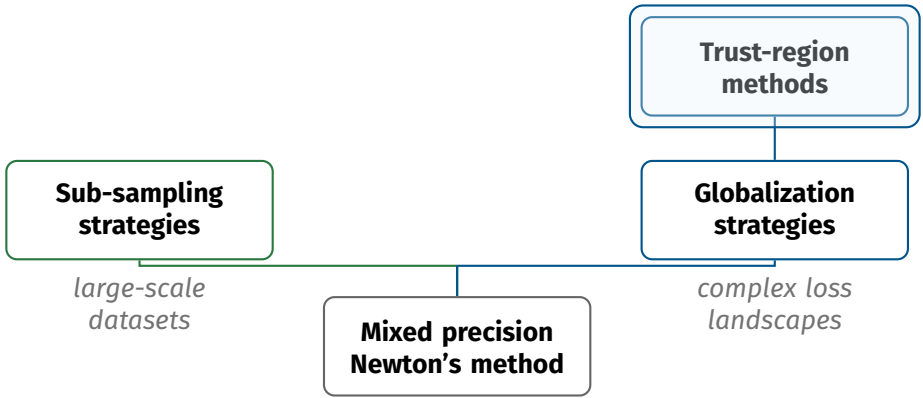

Trust-region methods

Model function $m_i(d) := f(\hat{x}_i) + g(\hat{x}_i)^T d + \frac{1}{2} d^T H(\hat{x}_i) d$

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Trust-region radius

Δ_i

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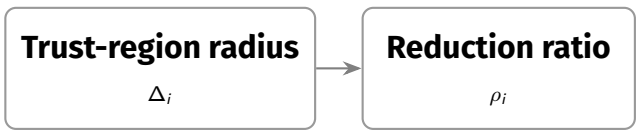

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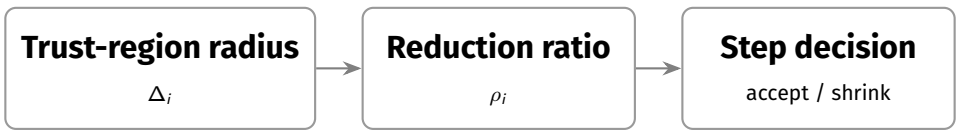

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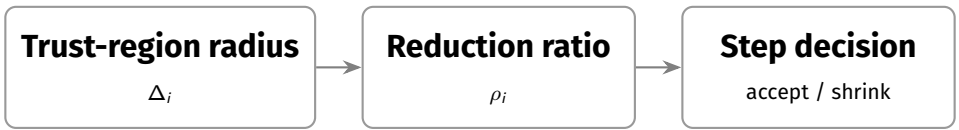

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Accept the step if ρ_i is large; otherwise shrink Δ_i and recompute.

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Binary classification task via **logistic regression**.

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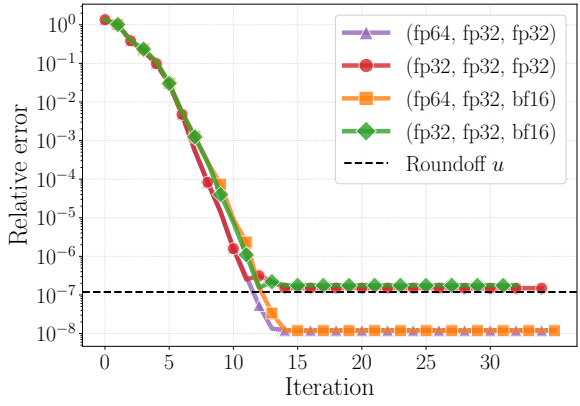
Australian Credit Approval dataset (UCI/Statlog)
690 instances, 14 attributes

Preliminary experiments



`Australian Dataset`. Mixed precision TR-Newton relative error.

Thank you!

Questions?

Gradient norm decrease

$$\|g(\widehat{x}_{i+1})\| \leq \phi_i \|g(\widehat{x}_i)\| + \psi_i.$$

Mixed precision Newton's method gradient norm decay.

- ϕ_i: "quadratic"/linear convergence influenced by Hessian errors;
- $\psi_i \approx \epsilon_i^g + \epsilon_i \|H(\widehat{x}_i)\| \|\widehat{x}_i\| \rightarrow$ **Potential stopping criterion!**

Convergence theorem assumptions

$$\|e_i^H\| \leq \epsilon_i^H \|H(\widehat{x}_i)\|, \quad \|e_i^g\| \leq \epsilon_i^g, \quad \|e_i^+\| \leq \epsilon_i(\|\widehat{d}_i\| + \|\widehat{x}_i\|).$$

At a given iteration

- $\epsilon_i^H \kappa(H(\widehat{x}_i)) < 1$

General local convergence

- $\epsilon_i^H \kappa(H(\widehat{x}_i)) < 1 \quad \forall i$
- $\theta_i := \alpha_i \|\widehat{x}_i - x^*\| + \beta_i < 1 \quad \forall i$
- until $\|\widehat{x}_i - x^*\| \leq \frac{\gamma_i}{1-\theta_i}$.

Numerical experiments problems

$$f(x) = 3 + \sum_{k=0}^{n-2} (x_k^2 + x_{k+1}^2)^2 - 4x_k.$$

ENGVAL1 function (from CUTEst dataset).

$$R(x) = x_0 z + \sum_{k=1}^{\lceil \frac{n-1}{2} \rceil} x_{2k-1} z^{k+1} + \sum_{k=1}^{\lfloor \frac{n-1}{2} \rfloor} x_{2k} \sin(x_{2k} z),$$

SINREG least squares residual, with z fixed data.

Gradient descent - Error model

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- τ_i: step size.

Convergence analysis of GD

Assuming f L-smooth, μ-strongly convex, and $\tau_i \in (0, 1/3L)$, we have

$$\|\widehat{x}_{i+1} - x^*\|^2 \leq \alpha_i^{GD} \|\widehat{x}_i - x^*\|^2 + \beta_i^{GD} \|\widehat{x}_i - x^*\| + \gamma_i^{GD},$$

Mixed precision gradient descent convergence rate.

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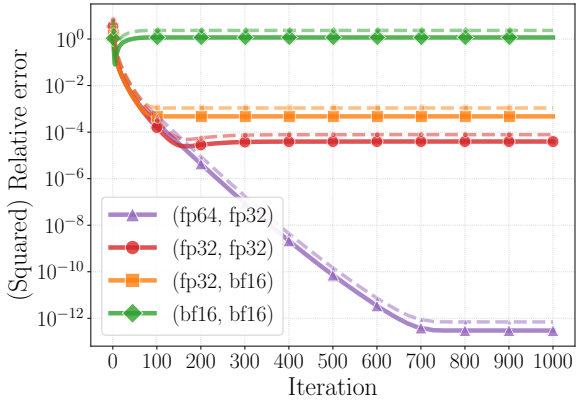
Observation: $\lim_{i\to\infty} \tau_i = 0 \implies \lim_{i\to\infty} \gamma_i^{GD} \approx u^2\|x^*\|^2$.

Preliminary experiments



`Quadratic function`. Mixed precision GD using finite differences and decaying τ_i.